Question 2

$$q(u) = \frac{u(2-u)}{2} \qquad q'(u) = \frac{2-2u}{2} = 1-u$$

$$\begin{cases} u_t + (q(u))_x = 0 \\ u(x,0) = g(x) \end{cases} \qquad g(x) = \begin{cases} 0 & x < 1 \\ 1 & x \in (1,2) \\ 2 & x > 2 \end{cases}$$

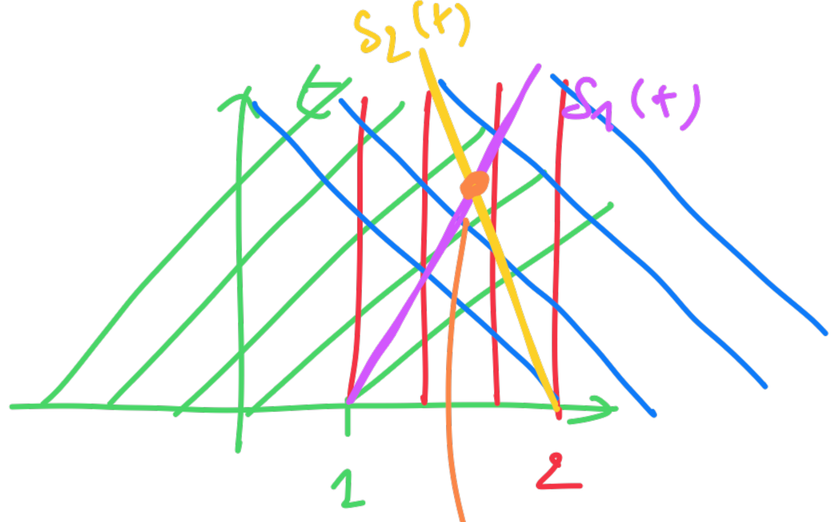
$$\frac{dt}{d\xi} = 1 \qquad \frac{dx}{d\xi} = (1-u) \qquad \frac{dz}{d\xi} = 0$$

$$t(0) = 0 \qquad x(0) = s \qquad z(0) = g(s)$$

$$t = \xi \qquad x = (1-g(s))\xi + s \qquad z = g(s)$$

characteristics given by $x(t) = (1-g(s))t + s$

$$x(t) = \begin{cases} t+s & s<1 \\ s & 1<s<2 \\ -t+s & s>2 \end{cases}$$



3rd possible shock

$$\dot{s}_1(t) = \frac{q(u_+) - q(u_-)}{u_+ - u_-} = 1/2 \qquad s_1(t) = t/2 + 1$$

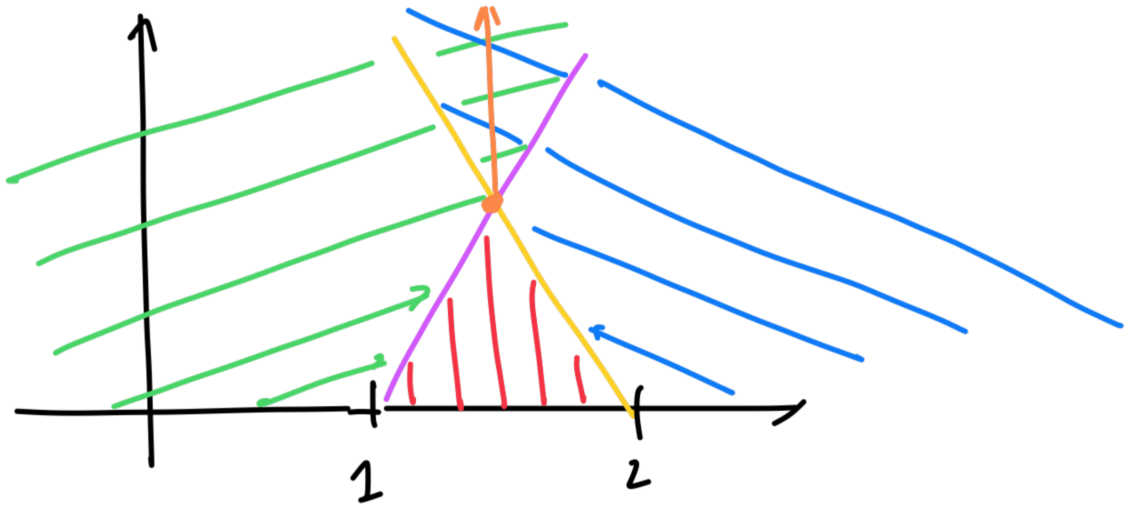
$$t = -\frac{1}{q''(g(x_0))\, g'(x_0)}$$

Then we look for a second shock

$$\dot{s}_2(t) = -\frac{1}{2} \Rightarrow s_2(t) = -\frac{t}{2} + 2$$

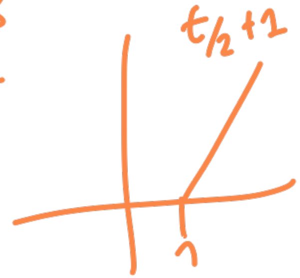
$t = 1$

$x = 3/2$



$\frac{t}{2}+1$ $\quad -t/2+2$

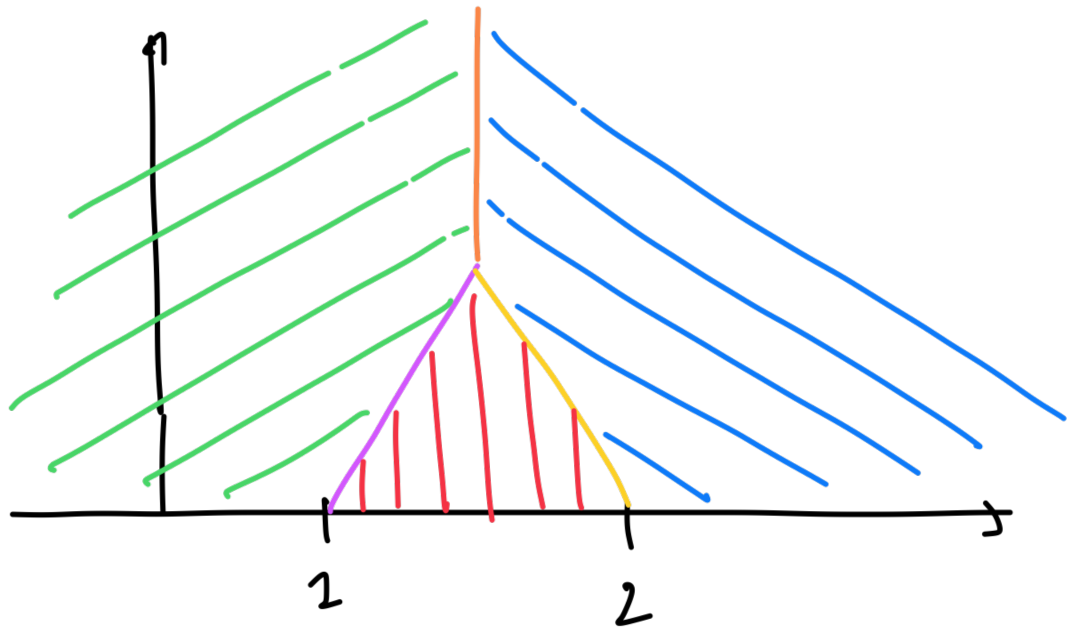
$\frac{3}{2}$ $\quad -3/4+\frac{8}{4}$



$5/4$

$u_+ = 2$ $\quad q(u_+) = 0$ $\quad \dot{s}(t) = 0$ $\quad s(t) = cst$

$u_- = 0$ $\quad q(u_-) = 0$ $\quad s(1) = 3/2 \rightarrow$


1
2

Question 6 a

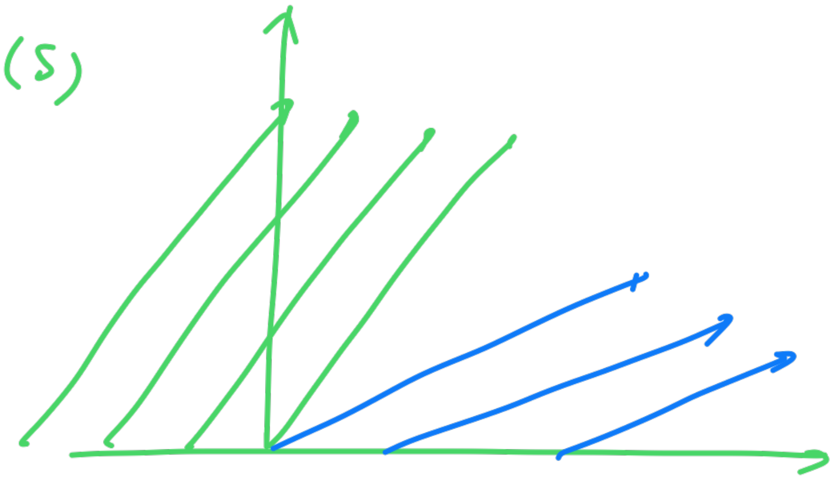
$$u(x,0) = g(x)$$

$$u_t + u^2 u_x = 0 \qquad g(x) = \begin{cases} -1 & x < 0 \\ 2 & x > 0 \end{cases}$$

$$\frac{dt}{d\xi} = 1 \qquad \frac{dx}{d\xi} = u^2 \qquad \frac{dz}{d\xi} = 0$$

$$t(0) = 0 \qquad x(0) = s \qquad z(0) = g(s)$$

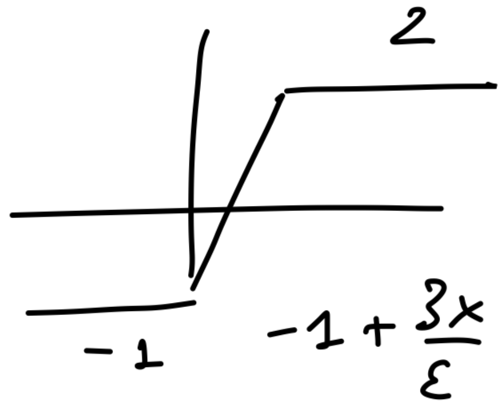
$$t = \xi \qquad x = g(s)^2 \xi + s$$

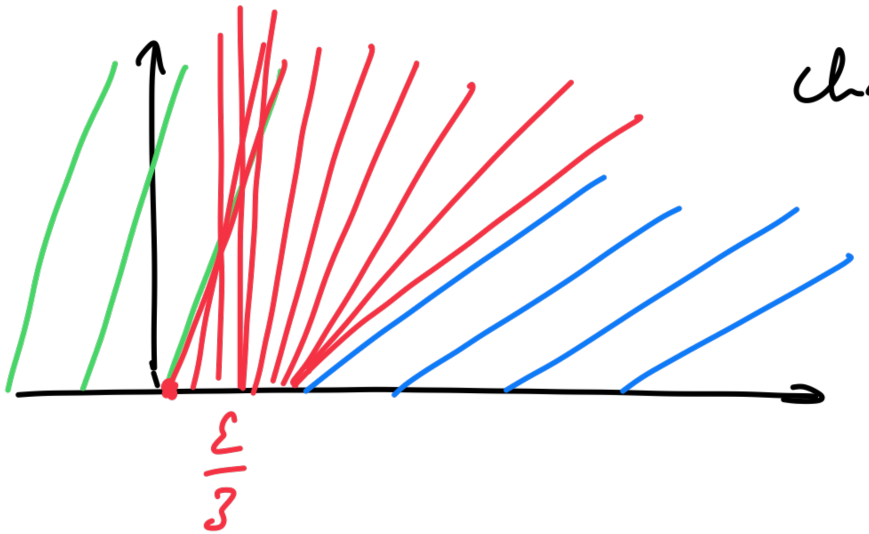


$$x(t) = \begin{cases} \xi + s & s < 0 \\ 4\xi + s & s > 0 \end{cases}$$

to introduce Fan like characteristics we consider the problem $u_t + u^2 u_x = 0$

with $g_\varepsilon(x) = \begin{cases} -1 & x<0 \\ -1+\frac{3x}{\varepsilon} & 0<x<\varepsilon \\ 2 & x>\varepsilon \end{cases}$





characteristics given by $\qquad x$

$$x(t) = g^2(x_0)t + x_0$$

$$x(t) = \begin{cases} t + x_0 & x_0 < 0 \\ \left(-1+\frac{3x_0}{\varepsilon}\right)^2 t + x_0 & 0 < x_0 < \varepsilon \\ 4t + x_0 & \varepsilon < x_0 \end{cases}$$

$$u_\varepsilon(x,t) = \begin{cases} -1 & x < t \\ \underbrace{-1 + 3(\ldots x)^{1/2}}_{} & x \ldots \end{cases}$$

$$x = t + \frac{9x_0^2 t}{\varepsilon^2} - \frac{6x_0}{\varepsilon} t + x_0$$

$$(x - t) + \frac{9t}{\varepsilon^2} x_0^2 + \left(-\frac{6}{\varepsilon} t + 1\right) x_0$$

$$\dot{s}(t) = \frac{q(u_+) - q(u_-)}{u_+ - u_-} = \frac{1}{3} \frac{\left(1 + \left(\frac{x}{t}\right)^{3/2}\right)}{\left(1 + \left(\frac{x}{t}\right)^{1/2}\right)}$$

$s(t)$

$\longrightarrow x_0 = (x)^{1/2}$