Introduction to optimisation

Recitation 04
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Question 1 *Consider the following linear program*

$$\begin{aligned} \max \quad & 2x_1 + 2x_2 + 3x_3 + x_4 + 4x_5 \\ s.t. \quad & 3x_1 + 7x_2 + 2x_3 + 3x_4 + 2x_5 \leq 40 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0 \end{aligned}$$

Solve the problem using the simplex algorithm.

Question 2 *Consider the following linear program*

$$\begin{aligned} \min \quad & -4x_1 - 3x_2 \\ s.t. \quad & x_1 + 2x_2 \leq 8 \\ & -2x_1 + x_2 \leq 5 \\ & 5x_1 + 3x_2 \leq 16 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Solve the LP using the simplex method.

Question 3 *Consider the following linear program*

$$\begin{aligned} \min \quad & -2x_1 - 5x_2 \\ s.t. \quad & -x_1 + 3x_2 \leq 2 \\ & -3x_1 + 2x_2 \leq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Graph the feasible set and then solve the LP using the simplex method.

Question 4 *Consider the following linear program*

$$\begin{aligned} \min \quad & -x_2 \\ s.t. \quad & x_1 - 2x_2 \leq 2 \\ & x_1 - x_2 \leq 3 \\ & x_2 \leq 3 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

1. *Solve the LP using the simplex method.*
2. *Does the LP have a unique optimal solution? If not, derive an expression for the set of optimal solutions*

Question 5 *Consider the following linear program*

$$\begin{array}{ll} \max & 2x_1 - 3x_2 + x_3 \\ s.t. & x_1 + 3x_2 + x_3 \leq 12 \\ & -x_1 + x_2 + 2x_3 \leq 6 \\ & -x_1 + 3x_2 \leq 9 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{array}$$

Question 6 *Consider the following LP,*

$$\begin{array}{ll} \max & 1.1x_1 + 1.2x_2 + x_3 \\ s.t. & 2x_1 + 2x_2 + 2x_3 \leq 10, \quad x_1 + 3x + 2 + x_3 \leq 10, \quad 4x_1 + x_2 + x_3 \leq 10 \\ & 3x_1 + x_2 + 3x_3 \leq 10, \quad x_1 + 2x_2 + 3x_3 \leq 10, \quad 3x_1 + 2x_2 + x_3 \leq 10 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{array}$$

Find the dual problem for this program.

Question 7 *Find the dual problems for the following linear programs*

$$(a) \quad \begin{array}{ll} \max & 7x_1 + 17x_2 + 17x_3 \\ s.t. & x_1 + x_2 = 8 \\ & x_1 + 4x_2 + 3x_3 \geq 14 \\ & 3x_2 + 4x_3 \leq 9 \\ & x_1 \leq 0, x_2 \geq 0, x_3 \textit{ unrestricted} \end{array}$$

$$(b) \quad \begin{array}{ll} \max & -7x_1 + 22x_2 + 18x_3 \\ s.t. & x_1 + 5x_2 + x_3 + x_4 \leq 8 \\ & x_1 + x_2 + x_3 \geq 14 \\ & 3x_2 + 4x_4 = 9 \\ & x_1 \geq 0, x_2 \leq 0, x_3 \textit{ unrestricted}, x_4 \geq 0 \end{array}$$

Question 8 *Determine the maximum value of* $18x_1 + 4x_2 + 6x_3$ *under the constraints*

$$3x_1 + x_2 \leq -3, \quad 2x_1 + x_3 \leq -5, \quad x_1 \leq 0, x_2 \leq 0, x_3 \leq 0$$

by looking at the dual problem.

References

[1] Pablo Pedregal, *Introduction to optimization*. Vol. 46. New York: Springer, 2004.

[2] Roy H Kwon, *Introduction to linear optimization and extensions with MATLAB*, CRC Press 2013.