

Introduction to optimisation

Recitation 03

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Question 1 Consider the following problem

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 1 \\ & x_1 \leq 3 \\ & x_2 \leq 2 \\ & x_1, x_2 \geq 0. \end{aligned}$$

(i) Write the problem in standard form

(ii) Find a solution using the simplex algorithm and draw, on the feasible set, the vertices that the algorithm is generating.

Question 2 Use the simplex method to solve the following problem

$$\begin{aligned} \max \quad & 2x_1 - x_2 + 2x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 10 \\ & x_1 + 2x_2 - 2x_3 \leq 20 \\ & x_2 + 2x_3 \leq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Question 3 Use the simplex method to solve the problem

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 + x_3 \\ \text{s.t.} \quad & 4x_1 + x_2 + x_3 = 30 \\ & 2x_1 + 3x_2 + x_3 \leq 60 \\ & x_1 + 2x_2 + 3x_3 \leq 40 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

	Model A	Model B	Model C
Assembling	2	2.5	3
Painting	1.5	2	1
Packaging	1	0.75	1.25

Table 1: Table used in Question 8

Question 4 Use the simplex method to solve the following problem

$$\begin{aligned}
 \max \quad & x_1 - x_2 + 2x_3 \\
 \text{s.t.} \quad & 2x_1 + 2x_2 \leq 8 \\
 & x_3 \leq 5 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Question 5 A farmer has a garden of 180 acres on which he would like to plant two types of crop : type I and type II. It takes him one day to water an acre of type I and two days to water an acre of type 2 and there are 240 days during which he has access to water. Moreover, it takes him .3 day to collect one acre of type I and .2 days to collect one acre of type II and there is usually only 30 days during which he can harvest the crops. Write the problem as a linear program and solve it using the simplex algorithm.

Question 6 Solve the following linear program with the simplex method

$$\begin{aligned}
 \min \quad & 3x_1 + 4x_2 + x_3 \\
 \text{s.t.} \quad & 2x_1 + x_2 + x_3 \geq 20 \\
 & x_1 + x_2 + x_3 = 15 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Question 7 Solve the following linear program with the simplex method

$$\begin{aligned}
 \max \quad & 15x_1 + 25x_2 + 10x_3 \\
 \text{s.t.} \quad & -6x_2 + 3x_3 \geq 12 \\
 & x_1 + 4x_2 + 2x_3 = 10 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Question 8 A company produces three types of bikes: Model A, Model B and Model C. The time required for each step in the assembly line is given in table 1. The time available for assembling, painting and packaging of the bikes is respectively given by 4006 hours, 2495 hours and 1500 hours. The profit on each of the three model is respectively given by \$45 (Model A), \$50 (Model B) and \$55 (Model C). Write the problem as a linear program and find the number of units of each model that has to be sold in order to maximize the profit.

Question 9 An investor has up to \$ 250,000 to invest in three types of investments. Type A pays 8% annually and has a risk factor of 0. Type B pays 10% annually and has a risk factor of 0.06. Type C pays 14% annually and has a risk factor of 0.10. To have a well-balanced portfolio, the investor imposes the following conditions. The average risk factor should be no greater than 0.05. Moreover, at least one fourth of the total portfolio is to be allocated to Type A investments and at least one fourth of the portfolio is to be allocated to type B investments. How much should be allocated to each type of investment to obtain a maximum return?

Question 10 Solve the following linear program using the simplex method

$$\begin{aligned} \min \quad & 2x_1 + 10x_2 + 8x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \geq 6 \\ & x_2 + 2x_3 \geq 8 \\ & -x_1 + 2x_2 + 2x_3 \geq 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

References

- [1] Pablo Pedregal, *Introduction to optimization*. Vol. 46. New York: Springer, 2004.