Introduction to optimisation

Recitation 02
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Question 1 *Suppose that Amazon wants to buy new servers to replace outdated ones and considers two options: the standard model costs \$350 and uses 350W of power, requires two shelves of server rack and can handle 1000 hits/min. Alternatively, the cutting edge model costs \$1600 and uses 500W of power but only requires one shelf and can handle 2000 hits/min. The available budget is \$37800 and Amazon has 44 shelves which can be used to store the new servers. Moreover the power at hand can reach 12,200 W. Write the problem as a linear program in order to determine the numbers x_1 and x_2 of each unit the company has to purchase in order to maximize the number of hits it can serve every minute.*

Question 2 *Tesla produces two models of cars: Model 1 and Model 2 (roadster). Model 1 requires 4 seats, 4 doors and 7 cameras. The faster model 2 only requires 2 seats and 2 doors and comes with 5 cameras. The German Tesla factory currently has 400 seats, 450 doors and a total of 270 cameras available. In case the factory would need more seats and doors, it can purchase aluminum whose cost is \$200 a block. Tesla can produce 10 seats, 15 additional doors and 12 cameras from each block of aluminum. The total cost of producing the Model 1 is \$5000 and the total cost of producting the roadster is \$6000. Finally the minimum number of cameras to produce is 1000 units per month. Knowing that Tesla wants to minimize its production cost, write the problem as a linear program.*

Question 3 *A flow network is a connected directed graph with a source (s) and a sink (t) and in which each edge e has non negative weights called its capacity* $c(e)$. *A flow f can be formally described as a set of non negative edge weights that satisfy the following rules:*

- *The* capacity rule*: The flow through an edge must be less than the capacity of the egde*
- *The* conservation rule*: wiht the exception of the source and the sink, the flow into a vertex must equal the flow out of it.*

The maximum flow problem *consists in finding the maximum flow size, that is to say the maximum amount of flow out of the source that satisfies the above rules. Write*

	Q1	Q2	Q3	Q4
H^+ demand (kg)	1,200	1,100	1,300	1,000

Table 1: Hydrogen quarterly consumption estimates for the year 2023

	B1	B2	B3	B4	B5	B6
Electricity (kWh)	\$0.177	\$0.181	\$0.173	\$0.163	\$0.181	\$0.15
Water	\$0.0015	\$0.0016	\$0.0013	\$0.0015	\$0.0017	\$0.0018

Table 2: Estimated Electricity (per kWh) and water (per liter) price forecast for the year 2023 (two month periods)

the maximum flow *problem as a linear program in which the variables are the edge weights $f(e)$.*

Question 4 *A company that sells hydrogen for the car industry is planning a new type of green hydrogen based on electrolysis. Table 1 below shows the quaterly demand (in kg) for 2023 as communicated by the customers. Table 2 gives the estimated electricity and water prices for the same year. Producing 1 kg of hydrogen requires 53 kWh. Moreover, the water requirement for this new type of green hydrogen is 9 L of H_2O per kg of H_2 produced. The electricity can be used over 4 consecutive months and the water can be used over 6 consecutive months (i.e. if the company buys electricity in January, it cannot use it in May). The company buys electricity and water at the beginning of each 2 months period. Moreover, the total expense for water cannot exceed 100 times the total expense for electricity.*

Question 5 ([1]) *Draw in the plane the region determined by the inequalities*

$$x_2 \geq 0, \quad 0 \leq x_1 \leq 3, \quad -x_1 + x_2 \leq 1, \quad x_1 + x_2 \leq 4$$

Find the point(s) where the following functions attain their maximum and minimum values:

$$2x_1 + x_2, \quad x_1 + x_2, \quad x_1 + 2x_2$$

Question 6 ([1]) *Solve graphically the next two problems:*

$$\begin{aligned} \max \quad & 2x_1 + 6x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 1 \\ & 2x_1 + x_2 \leq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} \min \quad & -3x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 5 \\ & 0 \leq x_1 \leq 4 \\ & 1 \leq x_2 \leq 6 \end{aligned}$$

Question 7 *Solve the following linear program*

$$
\begin{aligned}
\min \quad & 4x_1 + 5x_2 + 6x_3 \\
\text{s.t.} \quad & x_1 + x_2 \geq 11 \\
& x_1 - x_2 \leq 5 \\
& x_3 - x_1 - x_2 = 0 \\
& 7x_1 \geq 35 - 12x_2 \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0
\end{aligned}
$$

Question 8

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
s.t. \quad & x_1 + 2x_2 \leq 10 \\
& 2x_1 + x_2 \leq 16 \\
& -x_1 + x_2 \leq 3 \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

Question 9

$$
\begin{aligned}
\min \quad & x_1 + 3x_2 \\
& x_1 + 2x_2 \geq 6 \\
& x_1 - x_2 \leq 3 \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

References

[1] Pablo Pedregal, *Introduction to optimization.* Vol. 46. New York: Springer, 2004.