Introduction to optimisation

Recitation 01
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Question 1 *Lipton produces two types of beverages. Let us call them A and B. To do this, they purchase intermediate stage products in bulk and transform those into the final drinks A and B before distributing those drinks through various channels. The main constraint of Lipton is production capacity. Product A requires 3 machine hours per liter, but because of additional requirements, product B requires 4 hours of machine time per liter. The Lipton factory is limited to a total of 20,000 machine hours. The direct operating cost of product A is \$5 per liter while the operating cost for product B is \$3 per liter. Lipton's funds that are available to finance direct costs are set to \$40,000 in total. On top of this, the management expects that 40% of the sales of product A and 35% of the sales of product B can be collected during the production period and used to finance the operations. Product A is sold to the distributors for \$6 per liter while product B is sold to the distributors for \$4.50 per liter. The Lipton factory is currently closed because the production and marketing team on the one hand and the finance team on the other disagree on the spending of an additional \$250 for machine repairement. It has been estimated that such an additional expense could lead to an increase in capacity of 2000 machine hours but that the machines would have to be repaired again at the end of the period.*

- *Formulate this problem as a linear program in the two variables x_A and x_B, keeping in mind that Lipton's objective is to maximize the profit which corresponds to revenues minus the costs.*
- *Plot the set of feasible solutions (solutions x_A, x_B that satisfy the inequalities of your LP)*
- *Plot the isoprofit line $x_A + 1.5x_B = \$6000$. Is there any feasible production (x_A, x_B) that leads to a profit of \$6000? How could you find a production (x_A, x_B) that would maximize the profit?*

Question 2 *Turn the following LP into standard form*

$$\begin{aligned} \max \quad & 4x_1 - 2x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 2 \\ & x_1 - 2x_2 - x_3 \leq 1 \\ & x_1 + x_3 \geq -2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Question 3 *Find the solution of the following LP (using the Simplex algorithm)*

$$\begin{aligned} \min \quad & 3x_1 + x_2 + 9x_3 + x_4 \\ \text{s.t} \quad & x_1 + 2x_3 + x_4 = 4 \\ & x_2 + x_3 - x_4 = 2 \\ & x_i \geq 0. \end{aligned}$$