Analyse Numérique
Transformée de Laplace

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Question 1 *Compute the Laplace transform for the function*

$$f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 & t > 1 \end{cases}$$

Question 2 *Compute the Laplace transform for the following functions:*

$$(a) \quad f(t) = 4t, \quad (b) \quad f(t) = te^{2t}, \quad (c) \quad f(t) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}, \quad (d) \quad f(t) = 2\cos 3t$$

Question 3 *Without actually computing it, show that the following functions possess a Laplace transform*

$$(a) \quad \frac{\sin t}{t}, \quad (b) \quad \frac{1-\cos t}{t}, \quad (c) \quad f(x) \textit{ defined as in Fig. 1}$$

Question 4 *Find the Laplace transform of* $f(t) = te^{at}$ *and deduce the transform of* $g(t) = t^n e^{at}$

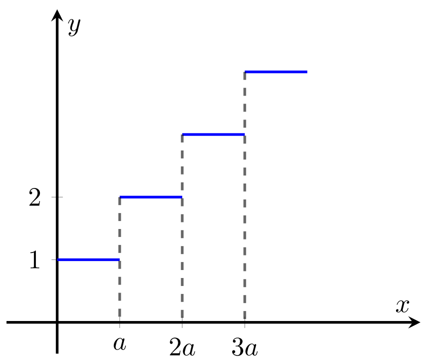

Figure 1: Question 3

Question 5 *For each of the following functions, determine which has a Laplace transform. if it exists, find it, if it does not, briefly say why*

$$(a)\quad e^{3t},\quad (b)\quad e^{t^2},\quad (c)\quad e^{1/t},\quad (d)\quad \frac{1}{t}$$

Question 6 *Find the Laplace transform for the following functions*

$$(a)\quad 4t+6e^{4t},\quad (b)\quad e^{-4t}\sin(5t)$$

Question 7 *Using the differentiation property, find the Laplace transform for the following functions*

$$(a)\quad te^{2t},\quad (b) t\cos t$$

Question 8 *Consider the function*

$$f(t)=\frac{\sin t}{t}=\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{(2n+1)!}$$

does this function admit a Laplace transform? Why? if yes, give the expression o the transform.

Question 9 *Using the properties of the Laplace transform, find the following inverse transform*

$$\mathcal{L}^{-1}\left(\frac{1}{2(s-1)}+\frac{1}{2(s+1)}\right)$$

Question 10 *Consider the Heaviside step function defined as*

$$H(t-a)=\begin{cases}1 & t>a\\ 0 & t<a\end{cases}$$

Compute the Laplace transform of $H(t-a)$ then deduce from it the inverse Laplace transform of $\frac{e^{-as}}{s}$

Question 11 *This time, we consider the indicator function on the $[a,b]$ interval.*

$$\mathbb{1}_{[a,b]}(t)=\frac{1}{b-a}\left(H(t-a)-H(t-b)\right)=\begin{cases}0 & t<a\\ \frac{1}{b-a} & a\le t<b\\ 0 & t\ge b\end{cases}$$

Find the Laplace transform of $\mathbb{1}_{[a,b]}(t)$

Question 12 *Using the translation property of the Laplace transform, and the fact that $\mathcal{L}(\sin\omega t)=\frac{\omega}{s^2+\omega^2}$, find the transforms of the following functions* *a)* $f_1(t) = e^{at}\cos\omega t$

b) $e^{at}\sin\omega t$

c) $e^{at}\cosh\omega t = e^{at}\frac{1}{2}\left(e^{\omega t} + e^{-\omega t}\right)$

d) $e^{at}\sinh\omega t = e^{at}\frac{1}{2}\left(e^{\omega t} - e^{-\omega t}\right)$

Finally use the above transforms to infer the inverse transform of $G(s)$,

$$G(s) = \frac{s}{s^2 + 4s + 1}$$

Question 13 *Use a partial fraction expansion to determine the inverse transform*

$$\mathcal{L}^{-1}\left\{\frac{a}{s^2 - a^2}\right\}$$

Question 14 *Determine the following inverse transform*

$$\mathcal{L}^{-1}\left\{\frac{s^2}{(s+3)^3}\right\}$$

Question 15 *Find the following inverse Laplace transforms*

a) $\mathcal{L}^{-1}\left\{\frac{s+3}{s(s-1)(s+2)}\right\}$

b) $\mathcal{L}^{-1}\left\{\frac{(s-1)}{s^2 + 2s - 8}\right\}$

c) $\mathcal{L}^{-1}\left\{\frac{3s+7}{s^2 - 2s + 5}\right\}$

d) $\mathcal{L}^{-1}\left\{\frac{e^{-7s}}{(s+3)^3}\right\}$