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NUMERICAL SIMULATION OF TIME-DEPENDENT BEACH AND DUNE EROSION

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ABSTRACT

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A computational procedure is developed for predicting the time-dependent, two-dimensional beach and dune erosion during severe storms due to elevated water levels and waves. The model employs the equation of sediment continuity and a dynamic equation governing the cross-shore sediment transport due to a disequilibrium of wave energy dissipation levels. These equations are solved numerically by an implicit, double-sweep procedure to determine the change in position of elevation contours in the profile. Given sufficient time, the profile will evolve to a form where the depth, h, in the surf zone is related to the distance seaward of the waterline by the relationship: $h = Ax^{2/3}$, which is consistent with many natural profiles and in which A depends on sediment characteristics.

The model is verified qualitatively and quantitatively through application to several idealized cases and through a preliminary simulation of erosion during Hurricane Eloise. In general, the time scales for shoreline response were found to be quite long relative to natural storm systems and erosion in the early response stages was found to be sensitive to storm surge height, but much less sensitive to wave height. The model response characteristics for simulation of erosion due to time-varying storm conditions show a lag between the maximum storm surge elevation and maximum erosion with the maximum erosion rate occurring at the time of the peak surge. For the simulated erosion due to Hurricane Eloise, reasonable agreement was found between the post-hurricane dune profiles and those calculated. However, the eroded volumes were in better agreement than the profile forms as the steepening of the natural dune profiles was not reproduced in the model.

INTRODUCTION

As attention is focused on the costs of mitigating erosion in coastal communities, it is increasingly important to develop reliable methods of estimat-

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ing both erosion rates and magnitudes associated with severe storms. In developed coastal areas, responsible individuals and agencies are confronted with a wide range of controversial alternatives for minimizing erosion damage, including coastal structures, beach nourishment, and setback or construction control lines. However, since engineering capabilities are not adequate to realistically predict erosion due to a specific storm event, the effectiveness of these remedial measures is difficult to evaluate.

Until more complete theoretical methods of quantifying sediment transport mechanisms in the surf zone are developed, one of the most promising approaches to the beach erosion prediction problem has been the use of schematic methods, based on the concept of ultimate profile equilibrium. Many investigators have recognized that beach profiles attain stable, equilibrium forms for given water level, wave height, and sediment characteristics (Bruun, 1954; Saville, 1957; Edelman, 1968, 1972; Swart, 1974; Dean, 1977). Bruun (1962) first advanced the argument that, where longshore sediment transport gradients do not exist, sand volume must be conserved along the beach profile such that beach erosion due to a slow sea level rise may be determined graphically, by shifting the equilibrium profile upward, an amount equal to the increase in sea level, and then landward, until the volume eroded from the beach face is equal to the volume deposited offshore to the closure depth.

In similar schematic arguments, Edelman (1968) and Dean (1976) proposed that short-term, storm-induced erosion, may also be determined graphically from a knowledge of pre- and post-storm equilibrium profile forms, the maximum storm surge level, and the breaking wave height. Edelman observed that during severe storms: (1) beach change is mainly the result of sand transport perpendicular to the shoreline; and (2) transfer of sand occurs from the berm and dune to the seaward edge of the surf zone where it is limited by incoming breaking waves. Using actual and idealized pre-storm profiles, Edelman established the known post-storm profile relative to the peak storm surge level, then shifted the profile landward to conserve sand volumes between the dune and the break point in a method similar to that used by Bruun. Dean (1976), using theoretical pre- and post-storm equilibrium profile forms, developed a solution for berm recession by analytically integrating and equating eroded and deposited volumes. In both methods, only the equilibrium erosion associated with the peak surge height is determined; the evolution of the profiles during the storm surge was not considered.

In a more detailed schematic method based on the analysis of large- and small-scale laboratory experiments, Swart (1974) defined a developing profile, including the surf and swash zones, in which both bed load and suspended load transport occur as a result of the dissipation of wave energy. Swart then established empirical expressions for the onshore and offshore limits of the active profile as well as for an equilibrium profile form. By assuming that the equilibrium form is eventually attained, Swart proposed that offshore sediment transport may be expressed in general terms, according to the disequilibrium of geometric profile characteristics. Thus, the transport rate, S_y, was expressed as a function of an empirically determined, time-dependent profile width, $(L_1 - L_2)_t$, as well as an equilibrium width, W, such that:

$$S_y = s_y[(L_1 - L_2)_t - W] \quad (1)$$

in which s_y is an empirical constant for given input conditions.

More recent experimental studies have considerably advanced the ability to reproduce dune erosion in the laboratory. Vellinga (1982, 1983) and van de Graaff (1983) have proposed a procedure that has been shown to agree reasonably well with measured post-storm profiles associated with the 1953 and 1976 events in Holland and Hurricane Eloise (1975) in west Florida. A reference profile is established for particular hydrographic and sediment characteristics. This result is then extended to "non-reference" conditions based on results from an extensive series of model studies. The method is empirical and strictly applicable only to a constant surge level over a five-hour duration; however, as stated by Vellinga, peak surge durations in excess of five hours may be accounted for on an "ad hoc" basis.

Hughes (1983) also proposed a scale relationship for physical models of beach and dune erosion. The relationship is based on equivalence of fall velocity parameter and ratio of inertia to gravity forces in model and prototype. The modeling requirements allow for model distortion and include a geomorphological time scale. The model relationships were evaluated against and compared favorably with dune erosion documented as a result of Hurricane Eloise in 1975.

Of the schematic erosion prediction methods, which include the work of Bruun, Edelman, Dean, and Swart, none is well verified or completely adequate for practical application in estimating storm-related erosion. Bruun's Rule assumes that nearly uniform deposition occurs over a considerable distance offshore beyond the breaker zone. While this theory has been largely substantiated for small water level increases over long periods, field observations after hurricanes or northeasters suggest that most significant sand deposition is limited to a short distance beyond the surf zone (Hayes, 1967). The Edelman and Dean methods, based on the assumption that the erosion potential of the maximum storm surge level is realized instantly, i.e. over the short duration of the peak surge, typically overpredict storm-related erosion (Chiu, 1977). Estimates of short-term erosion based on a steady-state maximum storm surge level do not account for time-dependent response of the beach and, in fact, represent the maximum erosion *potential* of the storm, a value that is seldom realized in nature. Although Swart makes significant advances by including a time-dependent sediment transport mechanism, this method neglects the role of dunes and remains too complex for widespread practical application. The methods of Vellinga, van de Graaff, and Hughes are the most realistic developed to date and include the effects of all relevant parameters including time-dependent variations in water level. However, the methods are empirical and are not easily applied generally to profiles or storm conditions that are far different from those specifically tested in the laboratory.

The present paper summarizes the development of an alternate schematic solution of beach and dune response to changing water levels and wave heights. This method differs from previous methods in that it is based on: (1) a numerical solution of simplified equations governing beach profile evolution, including a physically based mechanism defining the net cross-shore transport in the surf zone; (2) the complete time history of the storm surge, such that the time-dependent profile response is determined; and (3) a more realistic representation of the beach profile, including a composite-slope beach and dune form and a theoretical nearshore profile form which has been widely substantiated by several hundred field data sets. In addition, the proposed method relies on simplified input of basic surf zone parameters and it is easily applied to any open coast location for which sediment characteristics, profile forms, and the time-history of the storm surge are known.

THEORETICAL DEVELOPMENT

Based on an analysis of more than 500 beach profiles along the Atlantic and Gulf coasts, Dean (1977) found a general equilibrium profile of the form:

$$h = Ax^{2/3} \tag{2}$$

which, based on linear wave theory, was shown to be consistent with uniform wave energy dissipation per unit volume due to wave breaking. This expression, relating water depth, h, to distance offshore, x, is identical to the earlier empirical relationship found by Bruun (1954) for equilibrium profile forms. The scale parameter, A, was related theoretically to an equivalent equilibrium value of the energy dissipation per unit volume, D_*. Dean (1977), Hughes (1978), and Moore (1982) have analyzed over 700 beach profiles, from which an empirical relationship has been developed relating A (or D_*) to the mean sand grain diameter.

By assuming that the beach profile will eventually attain a dynamic equilibrium form for given water level and wave conditions, the net cross-shore transport rate may be assumed to be proportional to the disequilibrium of certain surf zone characteristics in a method similar to that used by Swart (1974). Based on equilibrium profile considerations, it is reasonable to express the offshore transport at any point in the surf zone, Q_s, in terms of the difference between the actual and equilibrium levels of wave energy dissipation in the surf zone; thus:

$$Q_s = K(D - D_*) \tag{3}$$

in which D is the actual time-dependent energy dissipation per unit volume and K is a transport rate parameter. It will be shown that with the equilibrium profile represented by Eq. (2), the energy dissipation per unit volume, D, can be expressed as a function of local water depth and bottom slope. Therefore, for a given point in the surf zone D increases as the water depth increases, i.e. during a storm surge, and Eq. (3) explains generally the resulting increase in offshore sediment transport. Conceptually, Q_s represents the combined bed and suspended load transport resulting from destructive forces during the breaking process, namely a level of wave energy dissipation which is above some dynamic equilibrium threshold at which no net transport occurs for given sand particles.

Swart (1976) found the transport parameter, s_y, to be a rather complicated empirical function of both sediment and wave characteristics. The parameter D in Eq. (3) accounts for sediment properties, profile shape, and assumed breaking wave characteristics. A study by Moore (1982) suggests that K may either be a constant or a weak function of other wave and sediment parameters. In a simulation of: (1) the large-scale laboratory experiments of Saville (1957) and (2) beach profile changes at Santa Barbara, California, based on measured wave characteristics over a one year period, Moore found K to be a constant, equal to 2.2×10^{-6} m^4/N. Based on dimensional arguments, the parameter K should vary with the length scale of the system. However, because Saville's tests were conducted at a fairly large scale, K appears to be applicable approximately to full-scale conditions.

The last requirement for the solution of time-dependent profile evolution is an expression for the conservation of sand over the profile. In the absence of longshore transport gradients, the equation ensuring conservation of sand in the shore-normal direction can be expressed as:

$$\frac{\partial x}{\partial t} = - \frac{\partial Q_s}{\partial h} \tag{4}$$

in which x represents the distance offshore to the contour, h, both Q_s and x are defined as positive in the offshore direction and h is positive below the still water level. If overwash is neglected, active profile change is bounded onshore by the wave runup limit or the region of active berm or dune erosion caused by undercutting and slumping of a steep sand face. Offshore, effective sediment transport is assumed to be effectively limited at the breakpoint of the incoming waves.

NUMERICAL SOLUTION

Finite-difference equations

For numerical application of Eqs. (3) and (4), the surf zone may be represented in schematic form as a series of uniformly spaced elevation contours, as in Fig. 1. With this definition, the undisturbed water depth, h_n, the departure of the water surface, η_n and the total depth, $h'_n = h_n + \eta_n$, may be

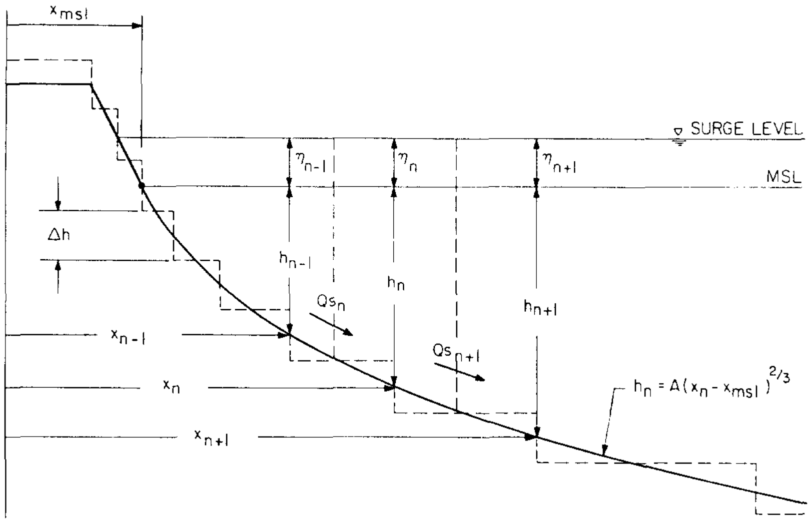

Fig. 1. Model representation of beach profile, showing depth and transport relation to grid definitions.

defined for n contours. The contour location, x_n, is then referenced to an arbitrary baseline in the dune. Employing a space-centered finite-difference scheme, the implicit form of the continuity equation may be written as:

$$\frac{\Delta x_n}{\Delta t} = - \frac{\overline{Q_{s_{n+1}}} - \overline{Q_{s_n}}}{\frac{1}{2}(h'_{n+1} - h'_{n-1})} \tag{5}$$

where $\overline{Q_s}$ represents the time-averaged sediment flux over the time step. In this initial model development, dynamic wave setup in the surf zone is not considered; therefore, a still water level is assumed where $\eta_n = \eta_{n+1}$ and:

$$\Delta x_n = \frac{\Delta t}{\Delta h} (\overline{Q_{s_n}} - \overline{Q_{s_{n+1}}}) \tag{6}$$

Introducing the transport equation, it follows that:

$$\Delta x_n = \frac{K \Delta t}{\Delta h} (\overline{D_n} - \overline{D_{n+1}}) \tag{7}$$

where $\overline{D}$ is the time-averaged wave energy dissipation per unit volume and a steady-state water level is assumed over the time step.

From a consideration of the change in wave energy flux over the surf zone width, the energy dissipation per unit volume may be expressed as:

$$D = \frac{1}{\overline{h}} \frac{\partial F}{\partial x} \tag{8}$$

According to shallow-water linear wave theory, the energy flux is:

$$F = \tfrac{1}{8}\gamma\kappa^2 g^{1/2} h^{5/2} \tag{9}$$

in which the breaking parameter, κ, is equal to 0.78 based on the spilling breaker assumption.

Substituting Eq. (9) into Eq. (8), the time-averaged energy dissipation per unit volume is given in terms of profile characteristics only, i.e. water depth and distance offshore, as:

$$\overline{D_{n+1}} = k_D \frac{h_{n+1}'^{5/2} - h_n'^{5/2}}{(h_{n+1}' + h_n')(\overline{x_{n+1}} - \overline{x_n})} \tag{10}$$

where

$$k_D = 1/4\gamma\kappa^2 g^{1/2}$$

Further substitution of Eq. (10) into the continuity equation, Eq. (7), results in the expression:

$$\Delta x_n = \frac{k_D K \Delta t}{\Delta h}\left[\frac{h_n'^{5/2} - h_{n-1}'^{5/2}}{(h_n' + h_{n-1}')(\overline{x_n} - \overline{x_{n-1}})} - \frac{h_{n+1}'^{5/2} - h_n'^{5/2}}{(h_{n+1}' + h_n')(\overline{x_{n+1}} - \overline{x_n})}\right] \tag{11}$$

where the interdependence of adjacent contours is evident through the appearance of $n - 1$, n, and $n + 1$ indices. By substituting the following expression for the time-averaged contour location:

$$\overline{x_n} = x_n + \frac{\Delta x_n}{2} \tag{12}$$

into Eq. (11), the onshore-offshore continuity equation can be written in the tridiagonal matrix form:

$$A_n \Delta x_{n-1} + B_n \Delta x_n + C_n \Delta x_{n+1} = Z_n \tag{13}$$

in which:

$$A_n = -\frac{\beta D_n}{x_n - x_{n-1}} \tag{13a}$$

$$B_n = 1 + \frac{\beta D_n}{x_n + x_{n-1}} + \frac{\beta D_{n+1}}{x_{n+1} - x_n} \tag{13b}$$

$$C_n = -\frac{\beta D_{n+1}}{x_{n+1} - x_n} \tag{13c}$$

$$Z_n = -\frac{2\beta}{K}(Q_{s_{n+1}} - Q_{s_n}) \tag{13d}$$

$$\beta = \frac{K\Delta t}{2\Delta h} \tag{13e}$$

In order to solve Eq. (13), it is necessary to introduce a recursion formula relating two adjacent unknowns:

$$\Delta x_{n-1} = E_n \Delta x_n + F_n \tag{14}$$

Substituting this expression into Eq. (13) establishes the new coefficients as:

$$E_{n+1} = \frac{-C_n}{B_n + A_n E_n} \tag{15a}$$

$$F_{n+1} = \frac{Z_n - A_n F_n}{B_n + A_n E_n} \tag{15b}$$

Since all A_n, B_n, C_n, and Z_n coefficients are known, a "double-sweep" implicit solution is employed. From an onshore boundary condition, E_n and F_n values are defined, from which all E_{n+1} and F_{n+1} coefficients may be calculated in an offshore sweep. Offshore beyond the breaking depth, no sediment transport or wave energy dissipation is defined and $\Delta x_n = 0$ in this region. In an onshore sweep, all Δx_{n-1} values are determined according to the recursion formula, Eq. (14).

Boundary conditions

To specify the onshore boundary conditions for the double sweep solution, the onshore profile representation must first be introduced. In most onshore-offshore erosion models, as well in beach planform models, composite berm-dune forms are neglected since theoretical expressions for swash zone and dune erosion mechanics are not available. However, for realistic estimates of storm-related erosion, at least a simplified dune erosion scheme must be developed.

In Figs. 2 and 3, the two berm-dune forms considered in this model development are presented. The dune form is represented schematically by a uniform dune height, h_D, and a linear dune face slope, M_D. A variable berm width, W_B, with either a distinct berm, or a break in slope at elevation, h_B, may be defined, along with a linear beach face slope, M_B. To ensure stability in the solution, the linear beach face slope intersects the concave equilibrium beach profile at a point of tangency, h_*, such that there is a smooth change in slope in the offshore direction.

With this basic profile definition, the double sweep solution is applied from h_* seaward to the breaking depth, h_b. This region is governed by the rate of energy dissipation per unit volume and may be termed the *dynamic* solution region. From h_* onshore, any number of solution schemes may be employed, depending on the desired level of complexity and accuracy. In this model, this region is defined as the *geometric* solution region and is governed by continuity only, since the energy dissipation levels are not defined above the water line.

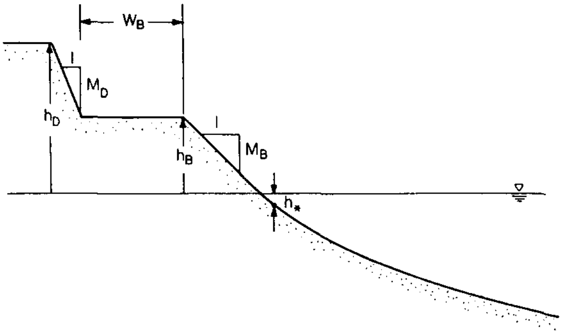

Fig. 2. Schematic dune representation with wide berm.

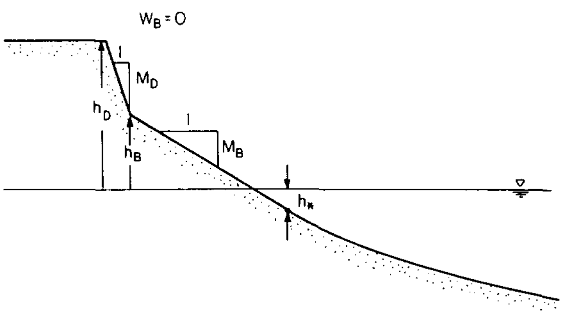

Fig. 3. Schematic dune representation with no berm.

In a more advanced model, a detailed swash zone transport relationship and variable upper limit on wave induced transport may be included to account for excess wave energy dissipated in the form of wave runup. For the present development, energy dissipation and sediment transport values are calculated at h_*, then the sediment transport curve is extended linearly to either the berm or dune crest where Q_s is set equal to zero. This upper limit of transport is determined according to defined criteria, and is analogous to coastal flooding models, where boundary cells are "opened" or "closed" depending on the water level in adjacent grids. Since the transport function is extended linearly to the berm or dune crest, the beach face and/or dune face are required to erode uniformly, thus maintaining the initial linear slopes above the still water level defined by the storm surge. Clearly this method does not simulate natural steepening of an eroding berm or dune; however, it does produce a first order estimate of erosion rates and magnitudes and it retains the essential characteristic of the dune as a reservoir of sand.

In carrying out the double sweep solution, as the water level is increased, the reference depths h_* and h_b are redefined relative to the new water level. With increased water levels, D and Q_s increase and the behavior of the governing equations is such that D and Q_s are maximum near h_* and decrease offshore. By extending the transport curve from h_* to zero onshore, it is possible to specify E and F coefficients on the beach face as $E_n = 0$ and $F_n = Z_n$ such that on the beach face $\Delta x_{n-1} = \Delta x_n$ since all $Q_{s_{n+1}} - Q_{s_n}$ are equal.

APPLICATION TO IDEALIZED CONDITIONS

General response characteristics

To demonstrate the characteristics of the numerical solution, consider an initial equilibrium profile with a linear beach face slope and distinct berm. This profile is then subjected to an instantaneous increase in the water level, which is maintained steady until the system regains equilibrium. With the increased water level and a constant breaking wave height, the surf zone width is suddenly decreased concentrating energy dissipation into a smaller volume nearshore. The system is in a state of disequilibrium, characterized by excess energy dissipation per unit volume and offshore sediment transport. In successive time steps, the beach face is eroded and material is deposited in the offshore regions of the surf zone, causing the surf zone width and volume to increase as the breaking position migrates offshore.

A plot of berm recession over time, in Fig. 4, reveals the characteristic solution to be nearly an exponential curve which approaches equilibrium asymptotically. In general, both the berm recession, R, and the volumetric erosion, V, may be approximated as:

$$R(t) = R_\infty [1 - \exp(-t/T_s)] \qquad (16)$$

or

$$V(t) = V_\infty [1 - \exp(-t/T_s)] \qquad (17)$$

where T_s is the characteristic time scale of the system. This behavior is consistent with observed erosion characteristics, both from large-scale laboratory tests by Saville (1957) and from a variety of small-scale laboratory tests by Swart (1974), Hughes and Chiu (1982), and Vellinga (1983a,b) among others. While this result is straightforward, it is significant as it indicates the importance of time in the erosion process. Numerical results indicate that time scales of natural beaches may be on the order of 10 to 100 hours for storm conditions and on the order of 1,000 to 10,000 hours when the effective limit of sediment motion is far offshore, as for sea level rise induced erosion or beach recovery following storms. Clearly, a given beach profile may require a considerable length of time to attain a new equilibrium form for given input conditions.

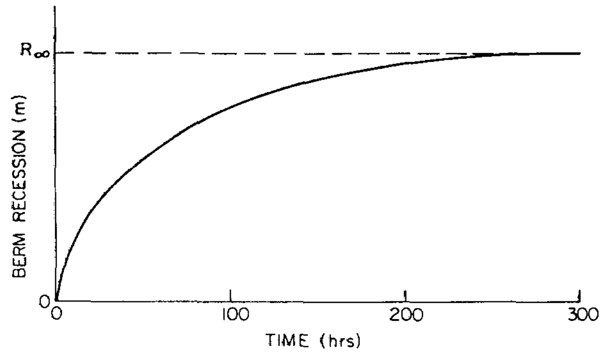

Fig. 4. Characteristic form of berm recession versus time for increased static water level.

In addition to demonstrating the importance and characteristic variation of temporal erosion effects, the ideal model response is useful in evaluating the effects of other parameters in the erosion model, including changes in water level, wave height, sand grain size, beach slope, and berm height. Consider a series of tests in which an equilibrium profile is subjected to various steady storm surge levels each with the same given wave height. If each water level is maintained as the profile approaches a new equilibrium, then, in Fig. 5, model results show an almost linear dependence between storm surge level and berm recession. In Fig. 6, although volumetric erosion does not follow the same linear dependence, it is clear that relevant time scales for erosion to equilibrium are nearly identical for each surge level, indicating that while water level is important in determining the total erosion potential of the system, it does not govern the characteristic time scale of response.

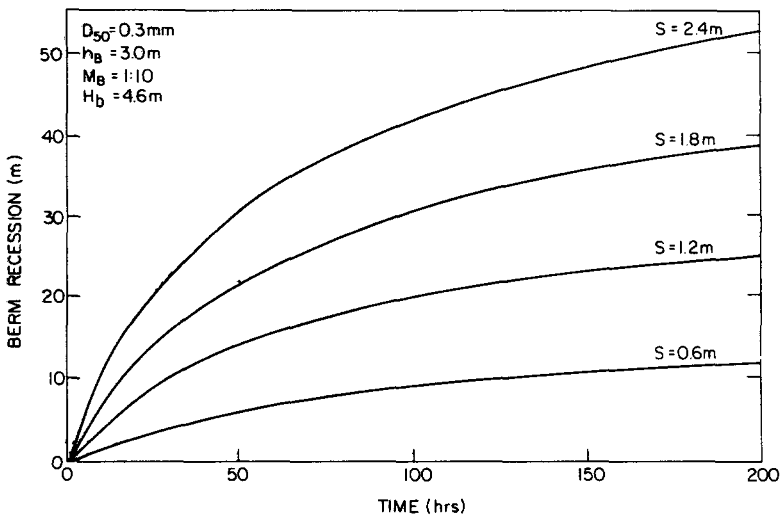

Fig. 5. Effect of static storm surge level on berm recession.

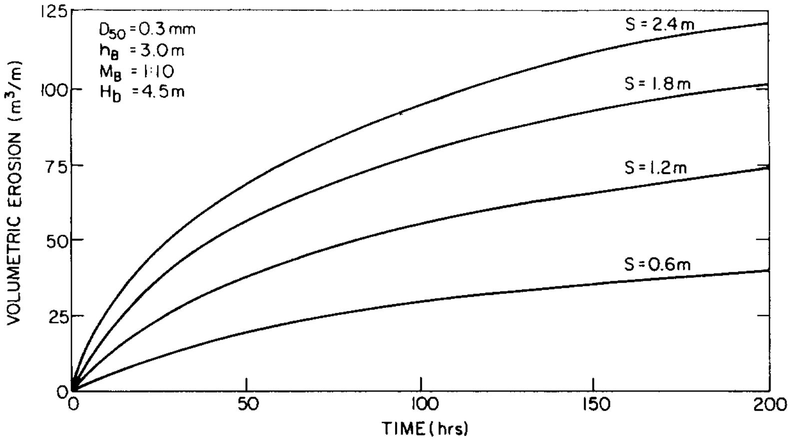

Fig. 6. Effect of static storm surge level on volumetric erosion.

In the counterpart to the previous case, the effects of different wave heights and the same steady storm surge on berm recession and volumetric erosion are presented in Figs. 7 and 8, respectively. The effect of increasing the wave height is to increase the surf zone width and the amount of sand that must be eroded to achieve equilibrium. However, since energy dissipation per unit volume is a function of water depth and bottom slope, erosion rates are not initially affected by a change in wave height since nearshore depths and slopes are initially the same in each case. In Figs. 7 and 8, the model solution is relatively independent of wave height during the early stages of the profile response and larger wave heights do not cause proportionally greater erosion. Therefore, since maximum storm surge durations are generally short and wave heights are typically large, say greater than 2.5 m,

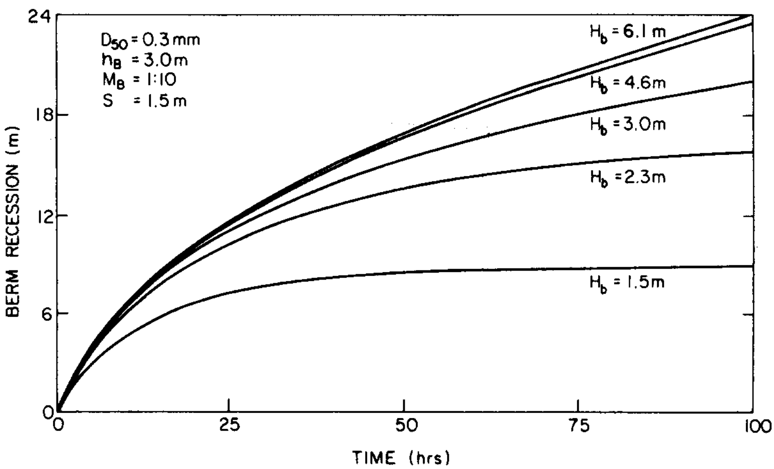

Fig. 7. Effect of wave height on berm recession.

an exact determination of wave height is not critical for the numerical model. It is interesting to note, however, that time scales of response to equilibrium increase dramatically with increases in wave height and it seems that the characteristic time scales are then related to characteristic length scales, such as surf zone width, which increase as the wave height, i.e. breaking depth, increases.

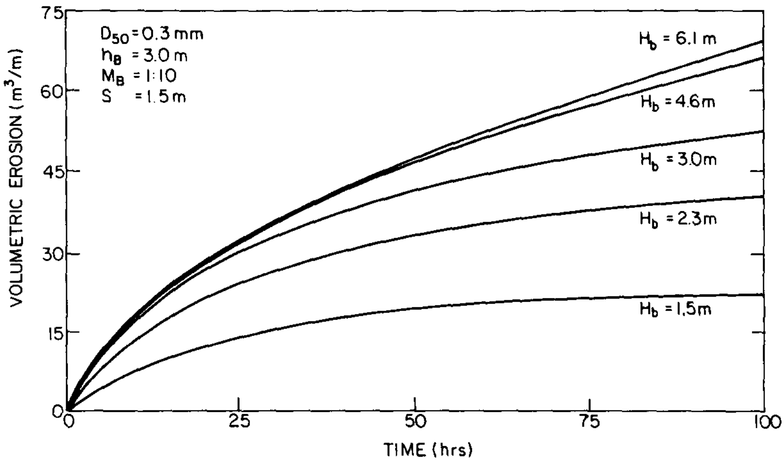

Fig. 8. Effect of wave height on volumetric erosion.

In nature, the relative importance of the water level versus the wave height, wave period, or wave steepness is difficult to determine. Surely an increase in water level alone, without appreciable wave energy, is not a cause of erosion. However, the increase in the water level is a vehicle for exposing upper portions of the profile to wave energy dissipation. In small scale laboratory tests with variable water levels, Hughes and Chiu (1981) have concluded that water level is the single most important forcing parameter in the storm surge-erosion process. Based on this observation, the numerical solution, in which the water level acts as the driving force and the wave height, through the breaking depth, serves as a boundary condition controlling the surf zone width, seems to agree qualitatively with laboratory results.

The model solution has also been found to agree qualitatively with field and laboratory results for the effects of sediment size. First, since the scale of the equilibrium beach profile is based empirically on sediment properties, through the equilibrium energy dissipation per unit volume, D_*, numerical results show a strong dependence on the sediment size. In Fig. 9, steeper beach profiles, with a larger sand grain size and larger A (and D_*) values, have shorter characteristic time scales and a smaller erosion potential than mildly sloping profiles with smaller sand grain sizes. It is possible to speculate that during severe storms, areas with a large median sand grain size may respond quickly to equilibrium and will therefore be relatively insensitive to storm duration, while equilibrium may not be attained for perhaps several days in areas of fine sand since sediment must be moved a greater distance to establish equilibrium.

In a final observation, when various beach face slopes are tested for a given sand grain size, in Fig. 10, model results show that profiles with steep beach face slopes erode farther and faster than profiles with milder beach slopes. In nature, the steep beach face represents a more unstable foreshore feature with a high erosion potential. Field observations of storm erosion have demonstrated this behavior (Chiu, 1977; Kana, 1977).

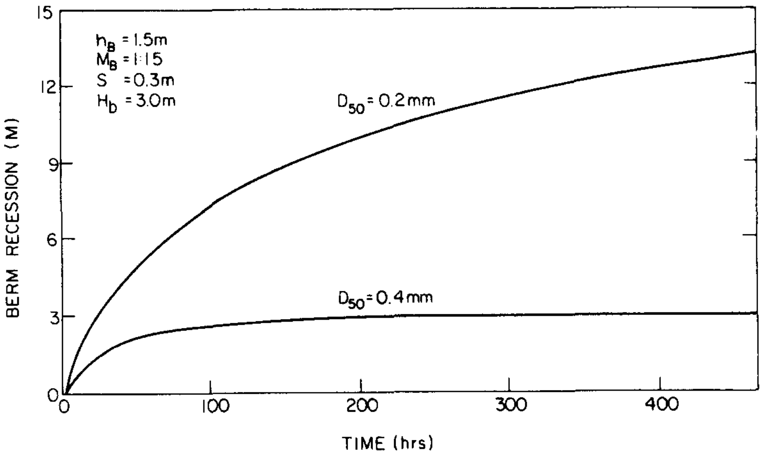

Fig. 9. Effect of sediment size on berm recession.

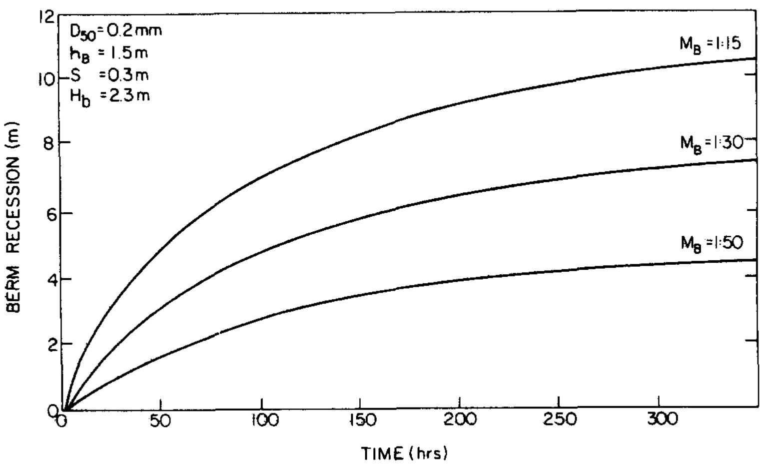

Fig. 10. Effect of beach face slope on berm recession.

Effect of storm duration

Prior examples have considered only ideal model behavior due to steady-state conditions, however it is of greater interest for practical application to simulate time-dependent erosion associated with changing water levels over the duration of a typical storm. Conceptually, to supply the input to the numerical model, a continuous storm surge hydrograph may be represented in stair-step fashion by a series of discrete water levels, such that steady-state conditions are maintained over each time step. As before, once the water level is established for each time step, the breaking depth, h_b, and the transition depth, h_*, are re-established relative to the new water level and the energy dissipation per unit volume and the sediment transport rate are calculated for the dynamic solution region. The resulting profile change over the time step can then be determined. By decreasing the time step, greater resolution is obtained in the solution; however, for most storm surge-erosion simulations, a time step of 1/2 hour seems to be a good compromise to reduce the number of computational steps.

To illustrate the ability of the present model to simulate the time-dependent profile response, an idealized storm surge hydrograph is input as:

$$\begin{aligned} \eta &= 1.2\cos^2\left[\frac{\sigma(t-18)}{2}\right], \quad |t-18| \leqslant T/2 \\ \eta &= 0, \quad |t-18| \geqslant T/2 \end{aligned} \tag{18}$$

in which $\sigma = 2\pi/T$ and T is the storm surge duration in hours. Together with a constant breaking wave height of 3 m, this idealized hydrograph is applied to a representative beach profile from Bay County, Florida. In Figs. 11, 12, and 13, the time-dependent erosion predictions are compared to the time history of the water level and the maximum erosion potential for storm surge durations of 12, 24, and 36 hours respectively. In order to compare the response of the model for various storm durations, the equilibrium or maximum potential erosion of the given profile is first determined for a steady-state peak surge height of 1.2 m. From this analysis, the maximum erosion potential of a 1.2 m storm surge for the given beach profile is 70.2 m^3/m and requires over 300 hours to be realized. With this maximum erosion value known, the solid curve in the figures represents the storm surge hydrograph, on the left axis, while the right axis is then scaled relative to the maximum potential erosion that would be realized if the profile responded instantly to equilibrium for the peak storm surge. The dashed curve, representing the numerical prediction for time-dependent profile response, reveals several interesting characteristics of the erosion process that, if representative of nature, can lead to an understanding of the dynamic beach response to a severe storm.

First, it is evident that the maximum erosion potential of the peak storm surge is not realized during rapidly varying storm conditions. From these examples, predicted maximum volumetric erosion for the given profile is only 14—28% of the maximum potential erosion; indicating that erosion is highly dependent on storm surge duration and that equilibrium is seldom attained during a typical hurricane-induced storm surge.

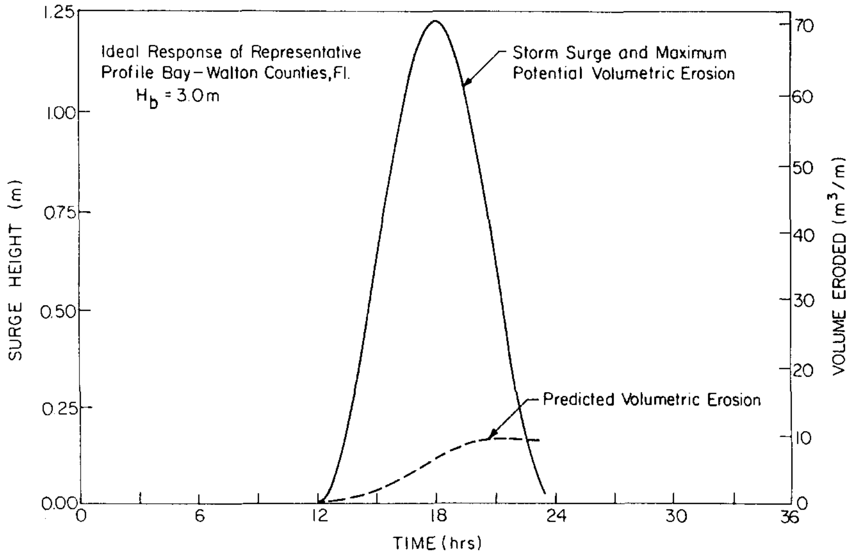

Fig. 11. Effect of storm surge duration on volumetric erosion, twelve hour storm surge duration.

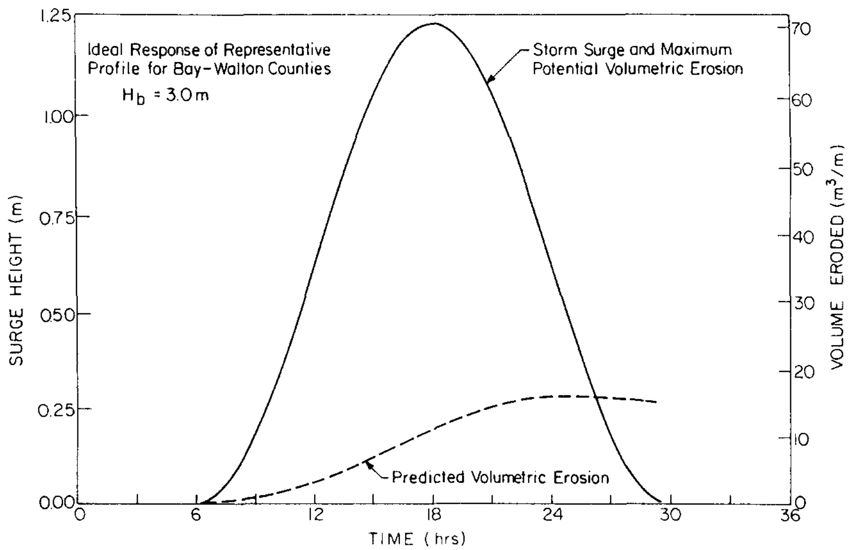

Fig. 12. Effect of storm surge duration on volumetric erosion, twenty-four hour storm surge duration.

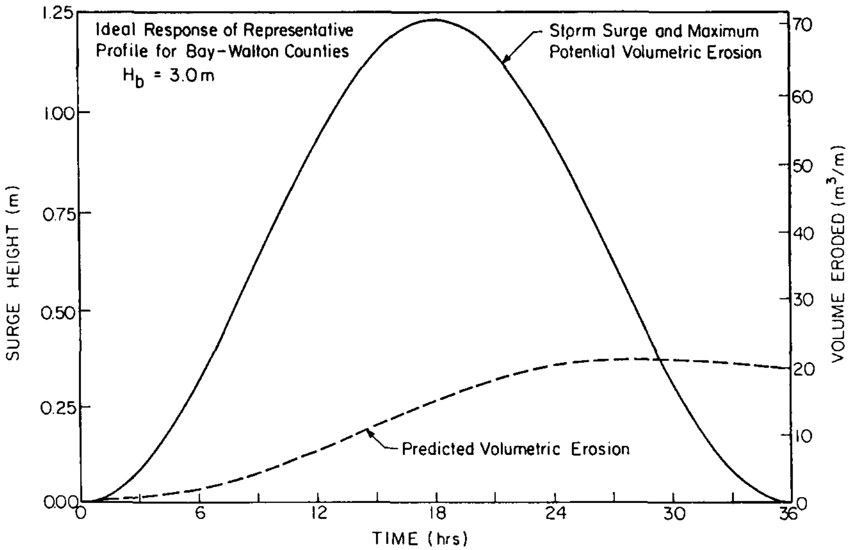

Fig. 13. Effect of storm surge duration on volumetric erosion, thirty-six hour storm surge duration.

The numerical results also indicate that the erosion rate is maximum at about the time of the peak storm surge. In nature a wide variety of dynamic conditions influence the exact beach response, including wave height, wave steepness, longshore and rip currents, nearshore wind-induced circulation, and bar formation. However, the present model shows that, in general, the dynamic erosion processes do not respond at the same rate as the increases in water level, thus the density of energy dissipation is maximum at the time of greatest difference between the potential and actual erosion values. Intuitively, since erosion proceeds at a relatively slow rate, for typical storm surge durations the peak storm surge represents approximately the condition of greatest disequilibrium in the profile. Based on the continuity equation, since $\partial Q_s/\partial h$ is maximum everywhere in the profile near the time of the peak surge, the erosion rate, $\partial x/\partial t$, is also maximum.

While it is widely believed that the maximum erosion magnitude must coincide with the peak surge height, the numerical solution indicates that erosion continues to increase for some time after the peak surge. As the water level begins to recede, the energy dissipation, sediment transport, and erosion rates also decrease as the system is brought back toward equilibrium. However, assuming that wave conditions continue to reflect storm conditions, the energy dissipation per unit volume in the surf zone is still greater than that which sand particles can withstand. Thus, the profile continues to erode. At some point, the water level is such that energy dissipation is near equilibrium over much of the profile, the sediment transport rate is essentially zero, and the erosion rate is also near zero. It is at this time, perhaps several hours after the peak surge level, that the maximum erosion is achieved. This lag between the peak storm surge and the maximum erosion has been documented in laboratory tests using variable water levels by Hughes and Chiu (1981) and Vellinga (1983a,b).

As the water level continues to decrease, the governing equations indicate that the energy dissipation per unit volume is less than the equilibrium value. If wave conditions continue to be destructive, erosion should cease as sand particles can now withstand the given level of wave energy dissipation. However, if wave conditions become constructive, as normally occurs after a storm, onshore transport may be initiated due to the predominant onshore bottom shear stress. While no effort has been made to determine the validity of the numerical solution in simulating beach recovery, the governing equations predict a reversal of sediment flux and the initiation of onshore transport immediately following the time of maximum erosion, such that the recovery rate increases as the water level decreases. As the water level approaches the original still water level, the system again experiences approximately its greatest disequilibrium, this time with an excess of sand in the profile; thus, the maximum recovery rate occurs at this time. This seems to agree with field observations which indicate that beaches begin to recover most rapidly immediately after a storm (Kana, 1977; Birkemeier, 1979). Calculations extended to encompass the recovery phase suggest that the calculated rate of recovery may be too rapid; however, it is possible that by recalibrating the transport parameter, K, to reflect constructive wave conditions, the model may be used to explain, in general terms, the entire erosion-recovery sequence.

EVALUATION OF METHOD BY HURRICANE ELOISE EROSION DATA

As a preliminary evaluation of the numerical model with prototype data, beach and dune erosion associated with Hurricane Eloise are calculated for Bay County, Florida. Hurricane Eloise crossed the Florida Panhandle on September 23, 1975 and was one of the most severe hurricanes ever experienced in that area. Although no tide gage records exist for the open coast of Bay County where the maximum storm surge occurred, it appears that the peak surge height was 2.4—3.7 m (Burdin, 1977; Chiu, 1977; Dean and Chiu, 1982). For erosion simulation, the storm surge hydrograph is reconstructed with a numerical storm surge model based on the Bathystrophic Storm Tide Theory of Freeman et al. (1957), calibrated according to the offshore bathymetry of the Bay County area. The resulting storm surge hydrograph is characterized by a rapid rise and fall of the water level as is typical of a fast moving hurricane and the peak surge height of 3.2 m agrees well with other numerical hindcasts.

Beach profiles for Bay County show natural variation in terms of dune height, beach slopes, and berm-dune configurations. However, Chiu (1977) and Hughes and Chiu (1982) give several schematic profile forms, from which a representative profile form may be defined. The initial pre-storm profile may be classified as a concave profile, with a high dune and no well defined berm. In Fig. 14, the representative profile has an average dune height of 5.2 m and a break-in-slope at a vegetation line at approximately 2.1 m. Average dune slopes are 1:2; average beach slopes are steep, typically 1:10 to 1:13. The offshore profile is characterized by the relationship:

$$h(x) = Ax^{2/3} \quad (19)$$

in which h is the water depth at a distance x from the intersection of the still water line with the beach profile. The equilibrium profile in Bay County is characterized by A = 0.13 $m^{1/3}$ which is empirically determined for 0.26 mm sand.

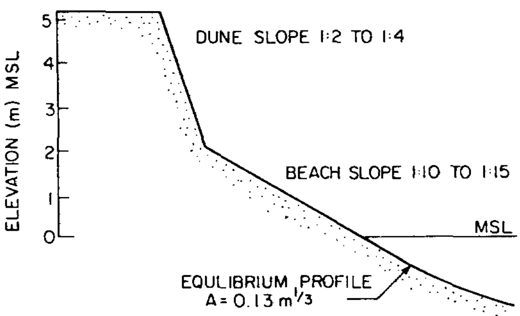

Fig. 14. Representative beach and dune profile, Bay-Walton County line.

For this initial model application, 20 test cases are simulated: (1) to evaluate the validity of the method relative to observed erosion; and (2) to display the sensitivity of the solution to changes in basic parameters. Since exact values of input parameters are not known, various combinations of two peak surge levels (3.2 and 3.5 m), three beach slopes (1:10, 1:13 and 1:15), two dune slopes (1:2 and 1:4) and two wave heights (2.3 and 4.6 m) are included.

In Table 1, the results of each test are summarized, such that the maximum erosion is shown in terms of volumetric erosion and in terms of contour change. It should be noted that the maximum eroded profiles are not in equilibrium but are in a state of transition toward equilibrium. For comparison, observed average erosion statistics presented by Chiu (1977) are listed in Table 2. Upon inspection, the range of calculated erosion magnitudes agree with observed values fairly well. First, consider beach-dune change as measured by the volumetric erosion. In the 20 test cases, volumetric erosion varies from 20.8 to 38.4 m^3/m, compared to average observed values of 18.3 to 20.4 m^3/m for Bay and Walton counties respectively and an average of 25.1 m^3/m near the area of the peak surge. While the predicted values are somewhat larger than observed, actual post-storm profiles reflect partial recovery of the beach face. Chiu (1977) notes the presence of a ridge of sand on the beach face and suggests that 5 m^3/m had been returned to the beach face at the time of the post-storm survey. Accounting for this additional sand volume, observed average erosion values would vary from 23.3 to 30.1 m^3/m.

From this discussion, it appears that the numerical solution accounts for time-dependent erosion in such a way that the magnitude of the eroded volume is in general agreement with field data of average volumetric erosion.

TABLE 1

Simulation of schematic profile changes for Bay-Walton Counties, Hurricane Eloise

Beach slope	Dune slope	Wave height (m)	Peak surge (m)	Advance/retreat for contour elevation				Average volume eroded (m^3/m)
				4.6 (m)	3.0 (m)	1.5 (m)	0 (m)	
1:10	1:2	2.3	3.2	−6.8	−6.7	−3.9	+7.0	27.1
1:10	1:2	4.6	3.2	−7.0	−6.9	−4.8	+4.2	30.1
1:10	1:4	2.3	3.2	−6.1	−6.3	−4.6	+6.8	26.6
1:10	1:4	4.6	3.2	−6.3	−6.5	−5.6	+4.0	29.9
1:13	1:2	2.3	3.2	−5.8	−5.8	−3.1	+4.1	23.1
1:13	1:2	4.6	3.2	−6.0	−5.9	−3.9	+0.9	26.1
1:13	1:4	2.3	3.2	−5.2	−5.4	−3.8	+4.0	22.8
1:13	1:4	4.6	3.2	−5.3	−5.5	−4.6	+0.9	25.8
1:15	1:2	2.3	3.2	−5.3	−5.3	−2.9	+2.3	20.8
1:15	1:2	4.6	3.2	−5.5	−5.4	−3.6	−0.6	24.1
1:15	1:4	2.3	3.2	−4.8	−4.9	−3.6	+1.8	20.8
1:15	1:4	4.6	3.2	−4.9	−5.1	−4.2	−0.9	24.1
1:10	1:2	2.3	3.5	−9.1	−8.6	−4.0	+9.1	33.4
1:10	1:2	4.6	3.5	−9.5	−9.0	−5.8	+4.9	38.4
1:10	1:4	2.3	3.5	−8.1	−8.4	−4.9	+8.9	32.9
1:10	1:4	4.6	3.5	−8.4	−8.8	−6.7	+4.5	37.9
1:15	1:2	2.3	3.5	−7.3	−6.8	−2.6	+3.7	26.1
1:15	1:2	4.6	3.5	−7.7	−7.1	−4.0	+0.3	30.9
1:15	1:4	2.3	3.5	−6.4	−6.7	−3.4	+3.3	25.8
1:15	1:4	4.6	3.5	−6.8	−7.0	−4.8	−0.5	31.1

TABLE 2

Observed erosion characteristics for Bay-Walton Counties (from Chiu, 1977)

Location	Advance/retreat for contour elevation				Average volume eroded (m^3/m)
	4.6 (m)	3.0 (m)	1.5 (m)	0 (m)	
Bay County	−2.7	−7.2	−1.8	+8.1	18.3
Walton County	−3.9	−10.7	−3.4	+6.9	20.4
Bay-Walton County Line (20-22 miles from Landfall)	−6.1	−12.8	−1.5	+12.8	25.1

To further illustrate the validity of the model in accounting for volumetric erosion due to variable water levels, the maximum *potential* erosion values obtained by allowing the numerical solution to reach steady-state equilibrium for the peak surge levels, are presented in Table 3. Comparing these results to either observed or predicted erosion, it is evident that the maximum erosion potential for the Hurricane Eloise peak storm tide is 5—10 times the amount realized from the time-dependent storm surge. Chiu (1977) notes that use of the Edelman (1972) or Dean (1976) schematic erosion prediction methods results in an overprediction of actual erosion by a factor of approximately five. Both of these methods consider erosion to occur to the potential consistent with the maximum storm tide. The maximum potential erosion, estimated by the steady state numerical solution is of the same order of magnitude as the graphical solution of Edelman and Dean. In contrast, when time-dependent storm surge levels are considered, predicted erosion seems to simulate nature in a much more realistic fashion.

Next, consider beach profile response as measured by contour advance or retreat. It is well known that an eroding berm or dune face steepens considerably, often approaching a vertical slope due to undermining and slumping as in Fig. 15. This behavior is quite evident in observed profiles from Bay and Walton counties, both in Table 2 and in other data presented by Chiu. In

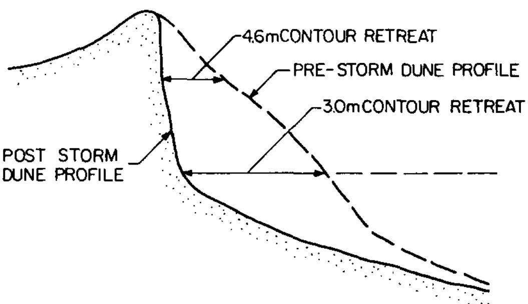

Fig. 15. Steepening of natural dune during erosion.

TABLE 3

Equilibrium recession for steady peak surge level. Schematic profile of Bay-Walton Counties

Beach slope	Dune slope	Wave height (m)	Peak surge (m)	Advance/retreat for contour elevation 4.6 (m)	3.0 (m)	1.5 (m)	0 (m)	Volume eroded (m^3/m)
1:10	1:2	2.3	3.2	−35.1	−33.5	+6.1	+50.3	112.9
1:10	1:2	4.6	3.2	−68.6	−65.5	−33.5	+21.3	263.5
1:10	1:2	2.3	3.5	−39.6	−33.5	+13.7	+48.8	120.5
1:10	1:2	4.6	3.5	−76.2	−70.1	−30.5	+29.0	288.6

the numerical model, on the other hand, it has been noted that the dune face is approximated by a linear slope which is maintained as the dune erodes as in Figs. 16 and 17. Therefore, in Table 1, the model solutions show a nearly equal recession of the 3.0 and 4.6 m contours; although there is a small difference since the peak surge level is slightly greater than 3 m. Since no attempt has been made to simulate dune steepening, present numerical results do not strictly match observed dune retreat profiles. However, it is interesting to note that because the volume of material eroded is essentially conserved, the uniform dune retreat represents an average retreat of all dune contours that compares favorably with the average recession of the 3.0 and 4.6 m contours in Table 2. Clearly, this simple averaging has little physical significance; however, it does suggest that the model results are of the cor-

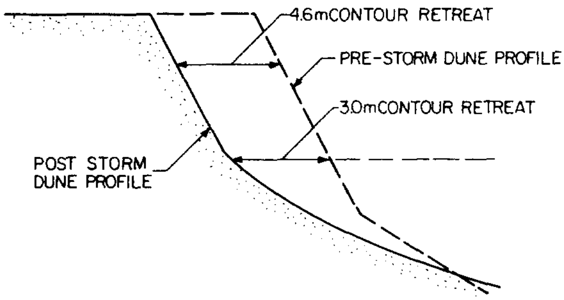

Fig. 16. Uniform recession of schematic dune form.

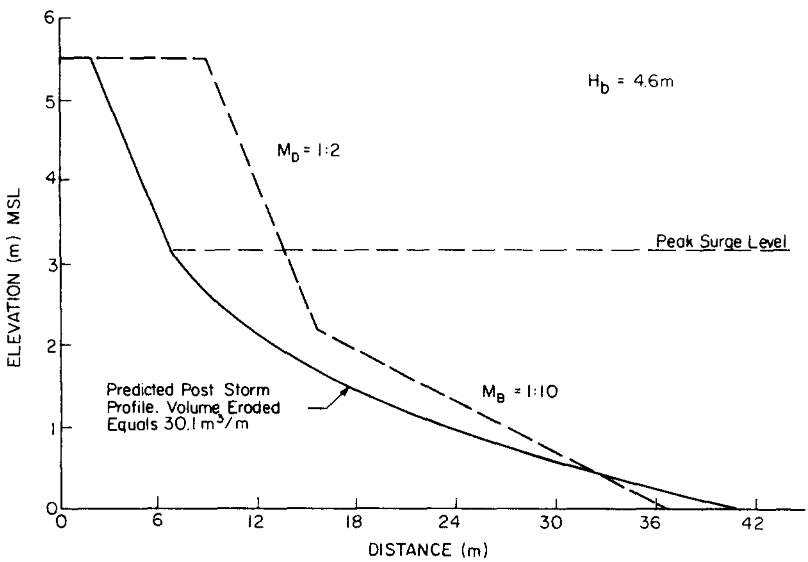

Fig. 17. Example of predicted post-storm beach-dune profile, Hurricane Eloise (1975), Bay-Walton County, Florida.

rect order of magnitude. This is especially evident when compared to the maximum potential erosion in Table 3 and Fig. 18, where dune recessions of 35.1—76.2 m are predicted for the 4.6 m contour after 500—1000 hours of simulation time.

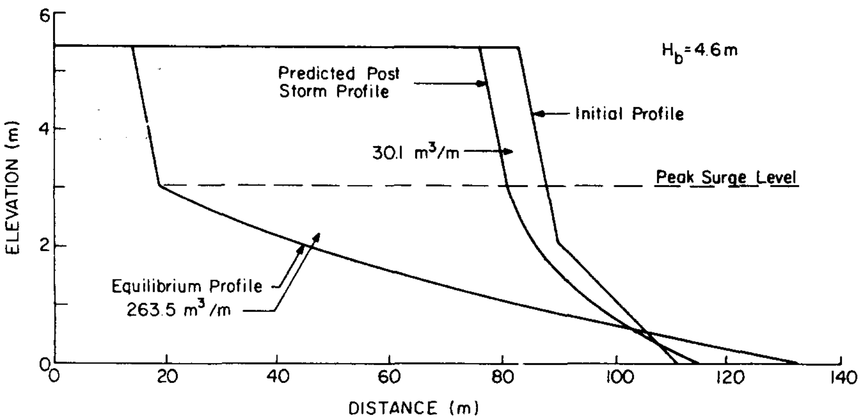

Fig. 18. Comparison of predicted post-storm beach-dune profile to maximum potential erosion associated with peak surge level.

Model sensitivities for time-dependent erosion simulation are similar to results discussed earlier with respect to water level and wave height effects. By increasing the storm surge level 9.5% over the duration of the surge, erosion during the storm simulation is increased by 23—26% for volumetric erosion and 34% to 38% for dune contour recession. For equilibrium conditions, in Table 3, volumetric erosion is increased by 6.7—9.5% while dune recession increases 11—13%. In contrast, an increase in wave height of 100% from 2.3 to 4.6 meters results in increased storm erosion of only 12—13% for volumetric erosion and 15—20% in dune recession. Equilibrium volumetric erosion values are increased 134—139% while 92—95% increases are evident for dune contour recessions. These results reinforce earlier conclusions that for short-term prediction, water level is the dominant forcing function while changes in wave height have less effect on storm induced dune erosion. For systems that approach equilibrium, however, model sensitivities to wave height do become quite important.

SUMMARY AND CONCLUSIONS

A formulation has been presented and incorporated into a numerical procedure to represent time-dependent beach profile response to varying wave heights and storm surge levels due to severe storms. The formulation combines the equation ensuring conservation of sand and a dynamic equation resulting from earlier studies of equilibrium beach profiles. The equations are cast into an implicit finite difference form and solved by a double-sweep procedure with suitable onshore and offshore boundary conditions. This method does not attempt to explain the detailed dynamics of the nearshore environment; rather, it satisfies a goal of simplicity, both in representing physical processes and for practical application.

A series of numerical solutions for idealized conditions have been carried out and form the basis for the following conclusions:

(1) The procedure is fairly detailed and efficient, accounting for prescribed time-varying wave heights and storm surge.

(2) The effect of storm tide on berm recession or volumetric erosion is much greater than wave height during the early stages of the response corresponding to durations of typical storms.

(3) Beaches composed of fine sand respond with longer time scales, and erode a greater distance than do beaches formed of coarse sand for the same forcing conditions.

(4) For the idealized cases modeled, the response time of the given natural beach-dune systems is so long that typical hurricane events may cause only 15—30% of the potential erosion associated with the peak storm surge.

For individual storm related erosion events, the proposed solution agrees qualitatively with many erosion characteristics observed in small- and large-scale laboratory experiments, as well as in the field. Quantitatively, numerical results compare favorably with observed beach-dune erosion data associated with Hurricane Eloise and with erosion prediction methods based on laboratory studies; notably allowing more realistic hindcasts of erosion magnitudes than other schematic erosion prediction methods.

Particular improvements to the present model should include:

(1) An inclusion of wave runup and dune erosion processes, such that dune steepening may be simulated.

(2) A realistic swash zone transport model to more rationally define sediment transport rates on the beach face.

(3) A more detailed description of wave breaking and bar formation, wave setup in the surf zone, or parameters to estimate the transition to and from an eroding profile.

(4) An improved quantification of the offshore sediment transport rate parameter to account for variability caused by wave or sediment characteristics.

(5) A capability to represent realistically the beach recovery phase and thus the cumulative effects of successive storms; and

(6) Extensive verification of the model with available prototype data from storm related erosion events, or with laboratory tests based on variable water levels.

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